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2. Mathematical physics

The results presented in Section 1 are applied to classical boundary value problems of mathematical physics such as the Dirichlet and Neumann problems for the Laplace equation. The $\mathbb{R}$-linear problem from Section 1 can be written as follows

$$u^+ = u^-,\ \frac{\partial u^+}{\partial n} = \lambda_k \frac{\partial u^-}{\partial n}\ on\ \partial D. \tag{1}$$

Here the function u is harmonic in D and $D^- := \cup_{k=0}^{n} D_k$, $\partial/\partial n$ is a normal derivative, λ_k are given constant. Note that Dirichlet and Neumann problems are limit cases of (1).

The method of functional equations is closely related to the generalized method of Schwarz [Mikhlin S.G., Integral equations, Pergamon Press, New York, 1964] which consists in replacing a given problem by a sequence of problems for a simple connected domain. The crucial ingredient in this approach is the repeated solution of the problems for the simple connected domain. In the previous works an additional condition on geometry of the domain was assumed to get a convergent algorithm corresponding to the generalized method of Schwarz. A modification of the generalized method of Schwarz has been proposed to get a convergent algorithm for arbitrary domain. Moreover, it is shown that the method of addition theorems can be considered as a discrete version of the generalized method of Schwarz. For details and other similar problems see Chapters 4 and 5 of [Mityushev V., Rogosin S. Constructive methods for linear and non-linear boundary value problems for analytic function. Theory and applications, Chapman & Hall / CRC, Monographs and Surveys in Pure and Applied Mathematics, NY etc, 2000, 283 pp.]